

Related Rates

1. $h = -2t^3 + 6t^2 + 10$

a) $v = -6t^2 + 12t$

$$v = -6(3)^2 + 12(3)$$

$$v = -6(9) + 36$$

$$v = -18$$

b) $a = -12t + 12$

$$0 = -12t + 12$$

$$t = 1$$

$$v = -6(1)^2 + 12(1)$$

$$v = -6 + 12$$

$$v = 6$$

c) $h = -6t^2 + 12t$

$$-6$$

$$-6$$

$$0 = -6t^2 + 12t - 6$$

$$0 = -6(t^2 - 2t + 1)$$

$$0 = -6(t - 1)(t - 1)$$

$$t = 1$$

2. $V = \frac{4}{3}\pi r^3$

$$r = 2 \text{ ft}$$

$$dr/dt = 3 \text{ ft/min}$$

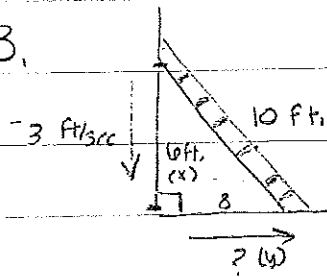
$$dv/dt = ?$$

$$dv/dt = 4\pi r^2 dr/dt$$

$$dv/dt = 4\pi(2)^2(3)$$

$$dv/dt = 48\pi \text{ ft}^3/\text{min}$$

3.



$$x^2 + y^2 = 100$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

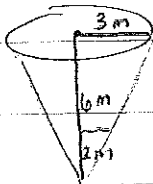
$$2(6)(-3) + 2(8) \frac{dy}{dt} = 0$$

$$-36 + 16 \frac{dy}{dt} = 0$$

$$16 \frac{dy}{dt} = 36$$

$$\frac{dy}{dt} = \frac{36}{16} = \frac{9}{4} \text{ ft/sec}$$

4.



$$dv/dt = 12 \text{ m}^3/\text{min}$$

$$h = 2 \text{ m} \quad \frac{dh}{dt} = ? \quad h = 2r \rightarrow r = \frac{h}{2}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{3} \pi \frac{h^2}{4} \cdot h$$

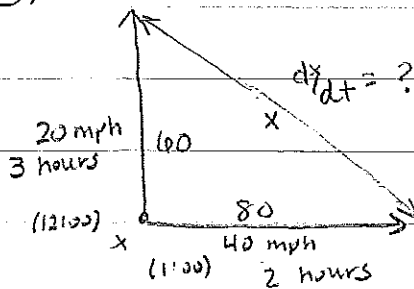
$$V = \frac{1}{12} \pi h^3$$

$$\frac{dv}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$(12) = \frac{1}{4} \pi (2)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{12}{\pi} \text{ m/min}$$

5.



3:00?

$$d = rt$$

$$d_A = (20)(3)$$

$$d_A = 60 \text{ mi}$$

$$d_B = (40)(2)$$

$$d_B = 80 \text{ mi}$$

$$(60)^2 + (80)^2 = X^2$$

$$X = 100$$

$$A^2 + B^2 = X^2$$

$$2A \frac{dA}{dt} + 2B \frac{dB}{dt} = 2X \frac{dX}{dt}$$

$$(60)(20) + (80)(40) = (100) \frac{dX}{dt}$$

$$4400 = 100 \frac{dX}{dt}$$

$$\frac{dX}{dt} = 44 \text{ mph}$$

6. $y = x^3 - x$

$$x = 2 \text{ cm} \quad \frac{dx}{dt} = 3 \text{ cm/sec}$$

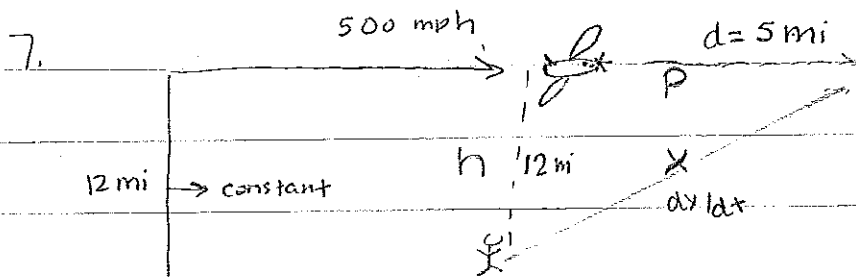
$$\frac{dy}{dt} = ?$$

$$\frac{dy}{dt} = 3x^2 \frac{dx}{dt} - \frac{dx}{dt}$$

$$\frac{dy}{dt} = 3(2)^2(3) - (3)$$

$$\frac{dy}{dt} = 36 - 3$$

$$\frac{dy}{dt} = 33 \text{ cm/sec}$$



$$d = rt$$

$$d = (500 \text{ mph})(.01)$$

$$30 \text{ sec} = .01 \text{ hours}$$

$$12^2 + 5^2 = x^2$$

$$d = 5 \text{ mi}$$

$$144 + 25 = x^2$$

$$x = 13$$

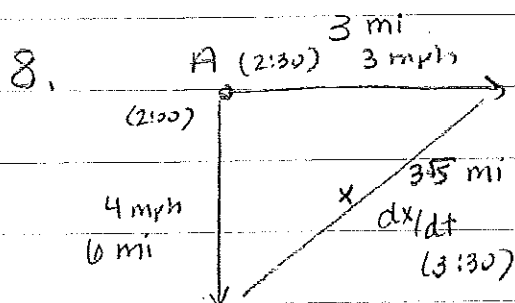
$$P^2 + H^2 = x^2$$

$$2P \frac{dx}{dt} + 0 = 2x \frac{dx}{dt}$$

$$2(5)(500) = 2(13) \frac{dx}{dt}$$

$$5000 = 26 \frac{dx}{dt}$$

$$\frac{2500}{13 \text{ mph}} = \frac{dx}{dt}$$



$$d_b = rt$$

$$d_m = rt$$

$$d_b = (3)(1)$$

$$d_m = (4)(1.5)$$

$$d_b = 3 \text{ mi}$$

$$d_m = 6 \text{ mi}$$

$$3^2 + 6^2 = x^2$$

$$B^2 + M^2 = x^2$$

$$9 + 36 = x^2$$

$$2B \frac{dx}{dt} + 2M \frac{dx}{dt} = 2x \frac{dx}{dt}$$

$$\sqrt{45} = \sqrt{x^2}$$

$$(3)(3) + (6)(4) = (3\sqrt{5}) \frac{dx}{dt}$$

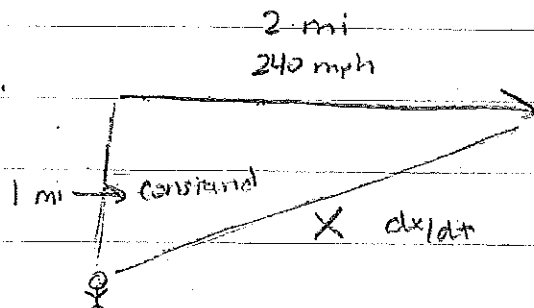
$$x = 3\sqrt{5}$$

$$9 + 24 = 3\sqrt{5} \frac{dx}{dt}$$

$$33 = 3\sqrt{5} \frac{dx}{dt}$$

$$\frac{11}{\sqrt{5}} \text{ mph} = \frac{dx}{dt}$$

9.



$$d = rt$$

$$d = (240)(.00833)$$

$$30 \text{ sec} = .0083333 \text{ hours}$$

$$d = 2 \text{ mi}$$

$$1^2 + 2^2 = X^2$$

$$1 + 4 = X^2$$

$$5 = X^2$$

$$X = \sqrt{5}$$

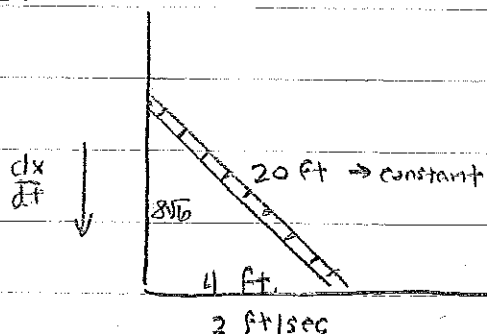
$$P^2 + H^2 = X^2$$

$$2P \frac{dP}{dt} + 0 = 2X \frac{dX}{dt}$$

$$(2)(240) = (\sqrt{5}) \frac{dX}{dt}$$

$$\frac{480}{\sqrt{5}} \text{ mph} = \frac{dX}{dt}$$

10.



$$X^2 + Y^2 = L^2$$

$$(X)^2 + (4)^2 = (20)^2$$

$$X^2 + 16 = 400$$

$$\sqrt{X^2} = \sqrt{384}$$

$$X = 8\sqrt{6}$$

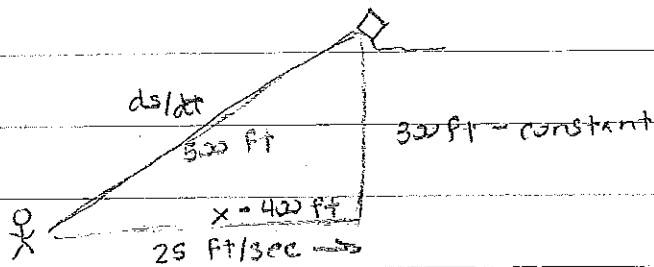
$$X^2 + Y^2 = L^2$$

$$2X \frac{dX}{dt} + 2Y \frac{dY}{dt} = 0$$

$$(8\sqrt{6}) \frac{dX}{dt} = -(4)(2)$$

$$\frac{dX}{dt} = \frac{-8}{8\sqrt{6}} = -\frac{1}{\sqrt{6}} \text{ ft/sec}$$

11.



$$(300)^2 + x^2 = (500)^2$$

$$90,000 + x^2 = 250,000$$

$$\sqrt{x^2} = \sqrt{160,000}$$

$$x = 400$$

$$x^2 + h^2 = s^2$$

$$2x \frac{dx}{dt} + 0 = 2s \frac{ds}{dt}$$

$$2(400)(25) = 2(500) \frac{ds}{dt}$$

$$10,000 = 500 \frac{ds}{dt}$$

$$\frac{ds}{dt} = 20 \text{ ft/sec}$$